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# ON EXPANDING $\frac{4}{n}$ INTO THREE TERM EGYPTIAN FRACTION

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***Abstract:*** **A fraction $(a/b)$ can be expressed as the sum of $N$ unit fractions. Such representations are known as Egyptian fractions. In general, each $\frac{a}{b}$ can be expressed by several different Egyptian fraction expansions. In this paper, we present a generalized formula for $4/n$ when N=3 for all positive integers $n$.**

***Keywords:*** **Egyptian Fraction, Unit Fraction, Shortest Egyptian Fraction.**

## I. INTRODUCTION

The Egyptian fraction $4/n$ of length three appeared to be an open problem in the field of number theory and a lot of researchers have paid attention to this problem. In particular, this is the motivating factor that leads to the idea of this work by attempting to investigate more. However, our efforts have some shortcomings due to the fact that, the expression for prime positive integers needs further research. It is vital to mention the assertion by Erdos in [1, 2] for $4/n$ which has not been implemented, i.e., 'Every Egyptian fraction can be express as a sum of three term unit fraction'. In this work, we proposed to examine the problem and enhance the conjectures asserted by Erdos in [1, 2]. This paper has been arranged as follows; we present brief overview and some basic definitions of Egyptian terminologies in section 2, the methodology of this work is given in section 3, and finally conclusion is presented in section 4.

## II. PRELIMINARIES (DEFINITION OF TERMS)

**Egyptian Fraction:** is the sum of distinct unit fractions, such as $\frac{1}{2}+\frac{1}{3}+\frac{1}{16}$. That is, each fraction in the expression has a numerator equal to 1 and a denominator that is a positive integer, and all the denominators differ from each other [3, 4, 5].

**Unit Fraction:** is a rational number written as a fraction where the numerator is one and the denominator is a positive integer. A unit fraction is therefore the reciprocal of a positive integer, $\frac{1}{n}$. Examples are $\frac{1}{1},\frac{1}{2},\frac{1}{3},\frac{1}{4}$ etc. [6].

**Shortest Egyptian Fraction:** A shortest Egyptian fraction for T/B where both T and B are positive integers is the shortest form of expressing T/B as a unit fraction. E.g.,

$$\frac{4}{9}=\frac{1}{5}+\frac{1}{95} \quad \text{and}$$

$$\frac{4}{13}=\frac{1}{4}+\frac{1}{18}+\frac{1}{468}$$

## III. THE EGYPTIANFRACTION OF $\frac{4}{n}$ OF LENGTH THREE

For the Egyptian fraction $\frac{4}{n}$ length three, more conjecture has been presented by [7, 8] on expressing $4/n$ as three-unit fraction:

However, we state the *simple formula for even denominators because:*

$$\frac{4}{2n} = \frac{2}{n} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{2n} \qquad (1)$$

*This formula (1) duplicates a fraction* $\frac{1}{2n}$*, which is not allowed in Egyptian expression. In addition, Gary Detlefs version has presented that, all the fractions must be unique, i.e:*

$$\frac{4}{2n} = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n(n+1)} \qquad (2)$$

*For multiples of 3, it was further stated as:*

$$\frac{4}{3n-1} = \frac{1}{n} + \frac{1}{3n-1} + \frac{1}{n(3n-1)}$$

$$\frac{4}{3n} = \frac{1}{n} + \frac{1}{4n} + \frac{1}{12n} \qquad (3)$$

In this work, we present all this conjecture in different version for all positive integers except prime denominators and finally stated its algorithm. We start by taken some cases:

**Case 1:**

Let $n$ be an even positive integer then

$$\frac{4}{n} = \frac{1}{\left(\frac{n}{2}\right)} + \frac{1}{\left(\frac{n+2}{2}\right)} + \frac{1}{\left(\frac{n}{2}\right)\left(\frac{n+2}{2}\right)}$$

**Case 2:**

Let $n$ be a positive prime number in the form of $3n - 1$ then

$$\frac{4}{n} = \frac{1}{\left(\frac{n+1}{3}\right)} + \frac{1}{n} + \frac{1}{n\left(\frac{n+1}{3}\right)}$$

**Case 3:**

Let $n$ be an odd positive integer multiple of three then

$$\frac{4}{n} = \frac{1}{\left(\frac{n}{3}\right)} + \frac{1}{(n+3)} + \frac{1}{(n+3)\left(\frac{n}{3}\right)}$$

**Algorithm 1: (Egyptian fraction for $4/n$)**

Step 1: Input $n$

Step 2: is $n > 4$?

If yes go to step 3;

Else go back to step 1;

Step 3: is $n \bmod 2 = 0$?

If yes set $\alpha = 1, \beta, \gamma = 0$ and go to step 6;

Else go to step 4;

Step 4: is $n \bmod 2 = 1$?

If yes set $\alpha = 0, \beta = 1, \gamma = 0$ and go to step 6;

Else go to back to step 5;

Step 5: is $n \bmod 3 = 0$?

If yes set $\alpha = 0, \beta = 0, \gamma = 1$ and go to step 6;

Else go to back to step 7;

Step 6: Compute

$$\frac{4}{n} = \alpha\left(\frac{1}{\left(\frac{n}{2}\right)} + \frac{1}{\left(\frac{n+2}{2}\right)} + \frac{1}{\left(\frac{n}{2}\right)\left(\frac{n+2}{2}\right)}\right) + \beta\left(\frac{1}{\left(\frac{n+1}{3}\right)} + \frac{1}{n} + \frac{1}{n\left(\frac{n+1}{3}\right)}\right) + \gamma\left(\frac{1}{\left(\frac{n}{3}\right)} + \frac{1}{(n+3)} + \frac{1}{(n+3)\left(\frac{n}{3}\right)}\right)$$

and go to step 7.

Step 7: Output $\frac{4}{n}$

## IV. CONCLUSION

In this paper we have reviewed [1, 2] conjecture and the general formula for $4/n$ for all positive integers $n$, was given except for the prime numbers in the form of $3n + 1$ which required further investigation. The fact that our formulas will converge as $n$ tend to be very large is possible, since the denominators are strictly increasing and stated by [9, 10], so we claim that our formulas are true for all positive integers $n$. Furthermore, if we have a solution for $^4/_n$ then we automatically has solutions for $4/nk$ for all k by multiplying each of the original denominators by k. This means that we need only to examine prime denominators in the form $3n + 1$.

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